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# MODAL ANALYSIS OF A LEAKY FEEDER CABLE MODELED BY A SHEATH HELIX WITH A CONDUCTING CORE

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ABSTRACT

Modes of a leaky feeder cable in tunnels and buried in earth are investigated by modeling the cable as a sheath helix with a conducting core. The numerical results are compared with previous work and used to plot dispersion diagrams.

## Introduction

Leaky feeders have been employed for some time now in communications systems in restricted or localized areas such as mines and road tunnels or along highways. Investigators concerned with leaky feeder systems have used various approximate techniques to gain insight into practical field problems.[1] The approach in this work is to model the braided sheath of a leaky feeder cable by a simplified structure, the sheath helix.[2,3,4,5] This useful model allows the rigorous formulation of the eigen-mode problem of the cable centered in a tunnel or buried in earth. The modes are found using numerical techniques and dispersion curves are plotted. Comparison of results are made with the results of Hill and Wait[6] for a cable with a helical sheath. The numerical results are shown to evolve logically from those of Neureuther, et al.[5] for the helical antenna on a conducting mast. The complex dispersion diagram allows a unified description of modes in terms of the general theory of open guiding traveling wave structures.[7]

A leaky feeder is usually a coaxial cable having for its outer conductor either a cylindrical shield with apertures or a loosely woven braid. The study of electromagnetic wave propagation along leaky feeders, if pursued rigorously, requires solutions of some very complicated boundary value problems in order to determine the external fields. The thin wire approximation along with the surface transfer impedance[1] concept has served well to reduce the amount of labor and provides results which agree qualitatively with available experimental results. The thin wire approximation is limited to the circumferentially invariant modes. Fortunately, however, these modes are usually the modes of primary interest since the higher order modes are more difficult to excite and are more lossy.

## Sheath Helix Model of Leaky Feeder

Another useful approach is to solve the full boundary value problem rigorously for a simplified structure. Here the simplified structure is a sheath helix insulated from a concentric perfectly conducting core. Hill and Wait[6,8] have shown that a cable using a helical shield has very similar characteristics as one having a loosely braided cable modeled by counter-wound helices. Furthermore it is well known that the sheath helix is a useful model for studying wave propagation along an actual helical structure provided the structure-period is much smaller than the modal period.[4] Historically the sheath helix model has been used first in the study of the traveling wave tube[2] and then to describe the characteristics of helical antennas.[5] Since the long braided cables employed in leaky feeder systems have transverse dimensions and a sheath structure much smaller than a wavelength along the cable axis, the sheath helix model should describe accurately the modes supported.

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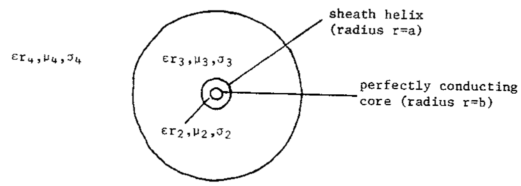


Figure 1. Cable situated in cylindrical media

## Numerical Solutions of Dispersion Equation

Figure 1 shows a cable situated in cylindrical media having in general different constitutive parameters. At each interface, equations expressing continuity of the tangential electric and magnetic fields are written. Resulting are eight equations in eight unknown field coefficients. A computer program has been written to solve this system of equations. The elements of the 8x8 matrix are evaluated using subroutines that generate the modified Bessel functions of complex arguments. After making an initial guess of the axial propagation constant $\beta_n$, a subroutine that uses Gauss elimination with pivoting and scaling calculates the matrix determinant. A value of $\beta_n$ which forces the determinant to a very small value is found after several iterations. After finding $\beta_n$ it is straightforward to solve the matrix equations and thus obtain the fields external to the leaky feeder cable. The process begins again when the frequency or material propagation constant is incremented.

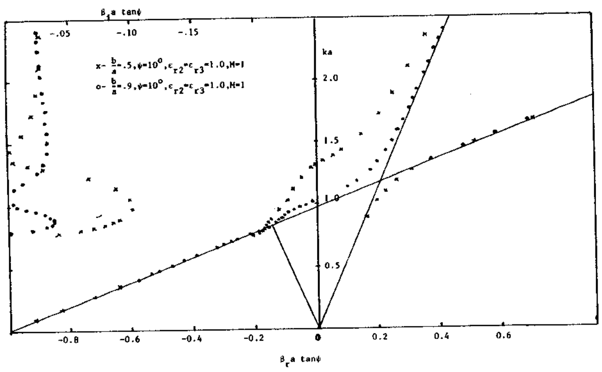


Figure 2. Dispersion curves of cable in free space.

## Results - Lossless Medium

In order to verify the operation of the programs the dispersion curves for a cable in an infinite lossless medium were plotted. Solutions corresponding to modes of a cable in free space with $\epsilon_2=\epsilon_3=\epsilon_0$, $\sigma_2=\sigma_3=0$ shown in Figure 2 agree perfectly with Neureuther's results. Also, plots of monofilar and bifilar modes of a cable in free space with $\epsilon_2=2.5\epsilon_0$ and $\epsilon_3=\epsilon_0$ shown in Figure 3 agree with those of Hill and Wait in the limit of a shield with infinitely many helical wires.

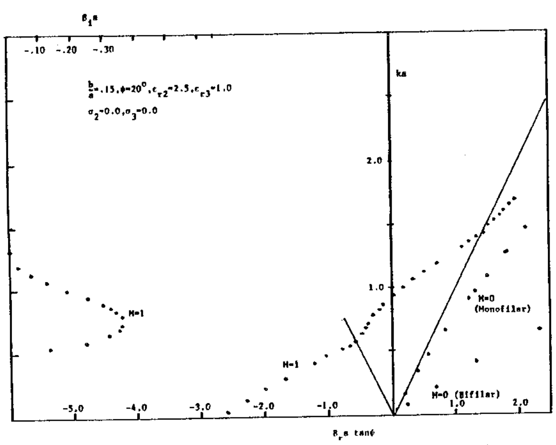


Figure 3. Dispersion curves of cable with dielectric constant $\epsilon_{r2}$=2.5 situated in free space

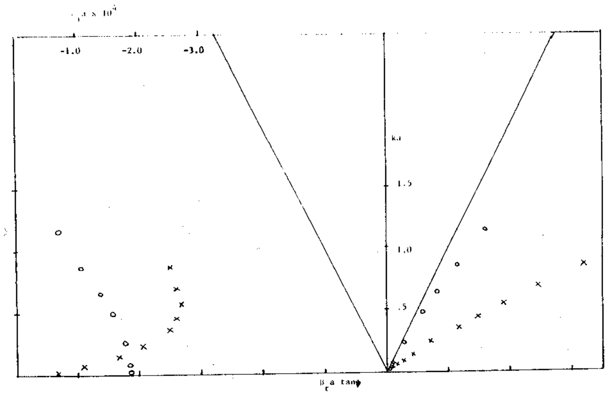


Figure 4. Monofilar and bifilar modes (M=0) of a cable in an infinite lossy medium.

## Lossy Medium

Figure 4 shows circumferentially symmetric (M=0) monofilar and bililar modes in a lossy medium having $\sigma_2=10^{-4}$ S/m and $\epsilon_2=\epsilon_0$ in order to show the effect of the medium conductivity on the modes. Except for an imaginary part which is small compared to that for complex modes in the visible region ($|\beta|<\kappa$), the dispersion curves are very much like those of Figure 2 for the structure situated in a lossless medium. Mode dispersion can clearly be seen in all cases where the cable dielectric is different from the surrounding medium. Also, more of the modal field propagates inside the cable as frequency increases. Both of the above effects are expected. It should be noted that the curves have been extended to higher frequencies which are more applicable to the helical antenna rather than the leaky feeder. Of course, it is realized that at extremely high frequencies the sheath helix model cannot be used since the modal period is no longer much larger than the period of the helix in the actual structure.

## Cable Buried in Earth or Situated in a Tunnel

Currently work is in progress on the modes of a cable located in a tunnel. In order to find monofilar mode solutions for this case initial estimates of the propagation constant must be very close to the actual value; otherwise it is found the solutions will inevitably converge to bifilar mode solutions. This behavior is interesting in view of the fact that Hill and Wait were able to find an analytic expression for the bifilar mode but not for the monofilar mode.

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